647510 **MSC4MatC404x** **Seat No :________**

M.Sc.(Maths.) Semester - 4 (*CBCS*) Examination
March/April - 2025 (NEW COURSE)
Graph Theory (Core)

Time: 2:30 Hours **Marks: 70**

Instructions:
1. All questions are compulsory.
2. Figures to the right indicate marks.

Que1(A) Answer any TWO of the following questions. **(10)**

(1) Show that in a given transport network G, the value of flow w from the source s to sink t is less than or equal to the capacity of any cut separating s from t.

(2) Let G be a graph and it does not contain any self-loop. Suppose for any pair of vertices $u, v \in G$, there is a unique path between u and v in G. Prove that G is a tree.

(3) Let G be a graph and it contains exactly two odd vertices, say $x, y \in V(G)$. Prove that x and y both lies in the same component of G.

Que-1(B) Answer any ONE of the following questions. **(04)**

(1) Define the following terms:
(i) Degree of a Vertex (ii) Loop
(iii) Null Graph (iv) Regular Graph

(2) Explain Maximal Flow in brief.

Oue-2(A) Answer any TWO of the following questions. **(10)**

(1) Let $G = (V, E)$ be a connected graph. Show that G is an Euler Graph if and only if degree of each vertex is even.

(2) Show that a connected graph G is an Euler graph if and only if G can be decomposed into edge disjoint cycles.

(3) State and prove Bondy & Chavatal's Theorem.

Que-2(B) Answer any ONE of the following questions. **(04)**

(1) Define the following terms:
(i) Euler Graph (ii) Unicursal Graph
(iii) Hamiltonian Cycle (iv) Closure of a Graph

(2) Explain Transport Network with example.

Que-3(A) Answer any TWO of the following questions. **(10)**

(1) Let T=(V,E) be a tree with n vertices. Show that $|E(T)| = n - 1$.

(2) Let u and v be two distinct vertices of a tree T. Show that there exists a unique path between u and v in T.

(3) Show that every tree has either one or two centres.

Que-3(B) Answer any ONE of the following questions. **(04)**

(1) Define the following terms:
(i) Tree (ii) Pendant Vertex
(iii) Acyclic Graph (iv) Minimally Connected Graph

(2) Show that if G is an acyclic graph with n vertices and k components, then $|E(G)| = n - k$.

Que-4(A) **Answer any TWO of the following questions.** **(10)**

(1) Define Edge Connectivity and Vertex Connectivity. Prove the following:

i. The edge connectivity of a graph G cannot exceed the degree of the vertex with the smallest degree in G.

ii. The vertex connectivity of any graph G can never exceed the edge connectivity of G.

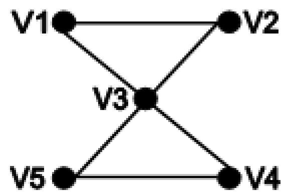
(2) Prove that Kuratowaski's first graph and second graph, both are non-planner graphs.

(3) Show that a connected planner graph G with n vertices, e edges has precisely $f = e - n + 2$ regions.

Que-4(B) **Answer any ONE of the following questions.** **(04)**

(1) Define Adjacency Matrix with an example.

(2) Find tree graph for the following graph



Que-5(A) **Answer any TWO of the following questions.** **(10)**

(1) Prove that every tree with at least two vertices is always 2-chromatic graph.

(2) Prove that the vertices of every planner graph can be properly colored with five colors.

(3) Show that a graph of n vertices is a complete graph if and only if its chromatic its polynomial is $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \cdots (\lambda - n + 1)$.

Que-5(B) **Answer any ONE of the following questions.** **(04)**

(1) Explain coloring of a graph with example.

(2) State and Prove *Max-Flow Min-Cut* Theorem.

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